

# A Weighted-Least Squares method for latency estimation in fMRI

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## Purpose

We developed an automated method for detecting and estimating latencies that takes advantage of what is empirically known about the hemodynamic response function.

## Introduction

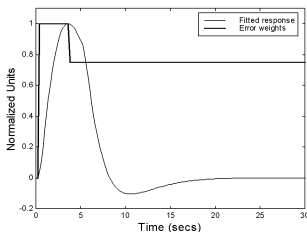
The utility of fMRI derives from the coupling between neuronal activity and hemodynamic factors. As such, it is reasonable to assume that there is information about the neuronal activity encoded within the hemodynamic response (HDR). There is empirical evidence that the HDR is a) time-locked to the stimulus within certain limits, b) less variable earlier in its evolution, and c) able to resolve information about relative hemodynamic timing [1-4]. Our algorithm weights more heavily the earliest part of the hemodynamic response, while still utilizing the entire data set. This has the additional advantage of providing a better estimate of latency for an unknown HDR.

## Theory

We model the fMRI signal as  $\mathbf{y} = \mathbf{X}_{\Delta t} \boldsymbol{\beta} + \mathbf{e}$  (where  $\Delta t$  is a discrete vector of latencies,  $\mathbf{y}$  is the data,  $\mathbf{X}$  is the model,  $\mathbf{e}$  is the error and all terms can be vectors), and fit in a weighted least squares (WLS) manner.  $\mathbf{X}$  is derived in the usual manner (convolving an ideal input vector or vectors with a canonical hemodynamic response function). The WLS fitting amounts to minimizing a cost function  $J = \mathbf{e}^T \mathbf{W} \mathbf{e}$  where  $\mathbf{W}$  is a diagonal weighting matrix with weighting as seen in Figure 1. The solution is straightforward and found to be  $\hat{\boldsymbol{\beta}}_{\Delta t} = (\mathbf{X}_{\Delta t}^T \mathbf{W} \mathbf{X}_{\Delta t})^{-1} \mathbf{X}_{\Delta t}^T \mathbf{W} \mathbf{y}$ . The latency is selected by choosing the  $\hat{\boldsymbol{\beta}}_{\Delta t}$  corresponding to the  $\Delta t$  which minimizes the multiple correlation coefficient,  $R$ . This is thus a sequential linear regression. We derive  $R$

to be: 
$$R_{\Delta t}^2 = \frac{J_{W_{o,\min}} - J_{W,\min}}{J_{W_{o,\min}}} = \frac{\hat{\boldsymbol{\beta}}_{\Delta t}^T \mathbf{X}_{\Delta t}^T \mathbf{W}^T \mathbf{y} - \frac{(\mathbf{y}^T \text{diag}(\mathbf{W}))^2}{\text{diag}(\mathbf{W})^T \mathbf{1}}}{\mathbf{y}^T \mathbf{W} \mathbf{y} - \frac{(\mathbf{y}^T \text{diag}(\mathbf{W}))^2}{\text{diag}(\mathbf{W})^T \mathbf{1}}}$$
, where

$J_{W_{o,\min}}$  is the cost function if the data are fit to a straight line (weighted by  $\mathbf{W}$ ), and  $J_{W,\min}$  is the cost function when  $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$ .



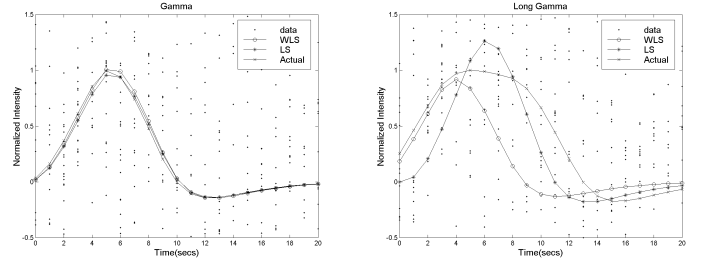
**Figure 1:** Weighting matrix for a given canonical hemodynamic response. The errors are weighted more heavily prior to the peak.

## Methods

We performed a simulated experiment and an actual experiment. The simulation consisted of an HDR embedded in independent Gaussian noise (SNR=1). We compared weighted vs. non-weighted sequential regression by calculating the resultant fit when the actual HDR is longer than the canonical HDR for each case. The actual experiment consisted of a visual-motor experiment in which the subject saw a 0.5 s, 8Hz full-field reversing checkerboard and subsequently pressed a response button. Scanning was performed on a 1.5T GE Signa scanner, using a GE-EPI (TR=1s, TE=39ms, FOV=24cm, matrix = 128x128, st=7mm, 9 slices) pulse sequence. All data were timing corrected and motion corrected prior to analysis [5].

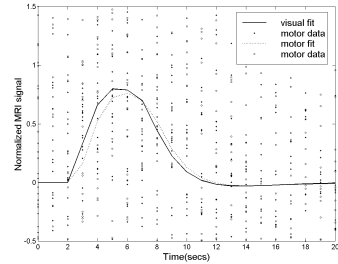
## Results

Figure 2 provides an example of our algorithm as compared with sequential regression analysis without weighting. Note that if the response is different from the canonical function the weighting forces the latency to be closely related to the onset of the function, even when it is extended in time.



**Figure 2:** Synthetic data (Gaussian, zero mean noise added to the hemodynamic signal) illustrating the tendency of the weighting matrix to better localize the onset of the HDR. Note the LS method finds a sort of mean latency.

The data from the visuo-motor experiment is demonstrated below. The single voxel with the earliest latency is plotted in Figure 3. Note that the visual response is earlier than the motor response, an intuitively appealing result.



**Figure 3:** Response from a single voxel in the visual cortex, and a single voxel in the motor cortex. We chose the voxel in each region corresponding to the earliest latency.

## Discussion

The fact that we can see a difference between motor and visual areas is encouraging. However, areas imputed in higher cognitive functions, involved in many feedback loops are likely to have activation patterns with complex temporal relationships. These relationships cannot be clarified unless one has a consistent and accurate way of comparing the latencies. Our WLS method provides a way to decrease sensitivity to shape differences and optimize latency comparisons.

## References

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